THE DIRECT ASYMMETRIC HYPERSONIC BLUNT-BODY PROBLEM

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ABSTRACT

The paper deals with numerical analysis of two-dimensional, steady, inviscid flow between a blunt body at an angle of attack and the detached shock wave. The so-called "direct" problem is investigated, i.e., the body shape, angle of attack, and free-stream conditions are assumed to be known, and the flow field as well as the shock shape are to be determined. The problem is treated by the method of integral relations, proposed by A. A. Dorodnicyn [1,2] and used first by O. M. Belocerkowski [3,4] in the symmetric case. In the present investigation, the problem is reduced to numerical integration of a set of four first-order ordinary differential equations with three unknown initial values, which are calculated by use of the relaxation method. The computations are performed for a prolate elliptical profile of the axes ratio $a/b = 4$, at free-stream Mach number $M_\infty = 3$, adiabatic exponent $\kappa = 1.4$ and for five angles of attack $\alpha = 0°$, $1°$, $2.5°$, $5°$ and $7.5°$. The relaxation method as elaborated by us turned out to be convergent in all the cases investigated.

INTRODUCTION

From the engineer's point of view it is often desirable to know the flow field around a flying object in the *whole* range of angles of attack, and not only for one particular value of this angle. In spite of this necessity, and due to rather serious mathematical difficulties of the problem, the bulk of the existing theoretical investigations of hypersonic flow around blunt bodies deals with the symmetric case, which corresponds to the symmetric two-dimensional profile, or to the axisymmetric body, both at the angle of attack equal to zero. The not very extensive research concerning the asymmetric case is usually restricted, with few exceptions, to small angles of attack, or to specific shock or body shapes. In the frame of the present investigation we attempted to develop a method that would be free of those restrictions, and suitable for computing the sub- and transonic flow field between a body of a prescribed shape and the detached shock wave, for any prescribed angle of attack.

This so-called "direct" problem was investigated in this paper under the following simplifying assumptions:

1. The body is two-dimensional, convex,* and its slope is continuous.
2. The flow is two-dimensional, steady, and rotational; the undisturbed flow is uniform and hypersonic.
3. The gas is inviscid, not heat-conducting; the real gas effects are not taken into account and the specific heats are constant.
4. The shock is governed by the Rankine-Hugoniot conditions.
5. The maximum entropy line wets the body, i.e., the stagnation streamline must intersect the shock at such a point, where it is normal.

To solve the problem stated, we applied in this paper the Dorodnicyn method of integral relations [1,2], which proved itself to be a very powerful tool in solving the direct hypersonic problem. The method was used successfully by Belocerkowski [3–8], Tshushkin [6,7,9,10], Holt [11,12], Traugott [13], and others for the computation of *symmetric* blunt-body flows, both two-dimensional and axisymmetric. Lately it was applied also to the *asymmetric* two-dimensional case by Vaglio-Laurin [14], by Bazshin [15,16] and by one of the present authors [17].† These papers [14–15], having much in common with our approach, should be briefly reported here.

Vaglio-Laurin [14] discussed the possibility of straightforward application of Dorodnicyn's method to the two-dimensional asymmetric case; he formulated the set of ordinary differential equations, to which our system is equivalent, and he proposed the above-mentioned condition (5), which—together with Dorodnicyn's conditions imposed on the solutions in the singular (saddle) points—allows the determination of the unknown

* This restriction is connected with the coordinate system s, n applied in this investigation (Fig. 1).

† Some of the results included in the present paper are already published [17].

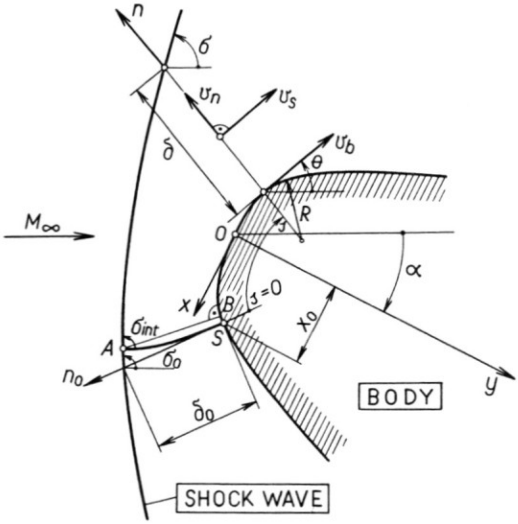

Figure 1. Nomenclature.

initial values. In this paper, as well as in both of Bazshin's papers [15, 16] the usual way [3–13] of straightforward numerical integration of the system of the said equations, is followed. Vaglio-Laurin's approach was different: he simplified the system and treated it by the *PKL* method. The interesting result presented in his paper [14] is the velocity distribution along a two-dimesional hypersonic profile with sonic shoulders.

Bazshin's paper [15] deals with a flat plate at four angles of attack. The form of the profile used simplifies the trial-and-error procedure to a great extent, because positions of both singular points are known. In his unpublished paper [16] Bazshin investigated the flow around a blunt symmetric profile, at Mach number $M_\infty = 5.8$ and at an angle of attack $\alpha = 30°$. The interesting and still unexplained feature of this case is a nodal point on the shock wave. The smooth "crossing" of this singularity serves as one of the conditions for determination of the unknown initial values.

SYMBOLS

GEOMETRICAL SYMBOLS

b	scale length (the minor axis of the elliptical profile)
n	coordinate; normal to the body
$R = (-ds/d\theta)$	radius of curvature
s	coordinate; a distance measured along the body

x,y	rectangular coordinate system, in which the body shape $y(x)$ is determined
x_0	coordinate of the stagnation point
α	angle of attack
δ	shock-wave distance
δ_0	"initial" shock-wave distance measured along the normal corresponding to the stagnation point
θ	angle between the tangent to the body and the direction of the uniform flow
σ	shock-wave angle; angle between the tangent to the shock wave and the direction of the uniform flow
σ_0	"initial" shock-wave angle corresponding to the stagnation point
σ_{int}	shock-wave angle in the intersection point of the stagnation streamline and the shock wave

Note: It is understood that n, R, s, x, y are nondimensional quantities, and that b is their scale.

GAS DYNAMIC QUANTITIES

$$a_\kappa = \sqrt{\frac{\kappa - 1}{\kappa + 1}} = \text{critical velocity}$$

$$g = \rho v_s^2 + kp$$

$$h = \tau v_n$$

$$H = \rho v_n^2 + kp$$

$$k = \frac{\kappa - 1}{2\kappa}$$

$$m = \frac{2 - \kappa}{\kappa - 1}$$

M_∞ = Mach number of the uniform flow

$$p = \tau^\kappa \varphi^{-1/(\kappa-1)} = \text{pressure}$$

$$t = \tau v_s$$

$v_b = w_b$ = velocity at the body

v_n, v_s = velocity components in the n and s direction, respectively

$$v_{n\delta} = w_2 \cos\theta - w_1 \sin\theta$$

$$v_{s\delta} = w_2 \sin\theta + w_1 \cos\theta$$

w = velocity modulus

$$w_\infty = M_\infty \sqrt{\frac{\kappa - 1}{2 + (\kappa - 1) M_\infty^2}}$$

$$w_1 = w_\infty (1 - \frac{2 \sin^2 \sigma}{\kappa + 1} \lambda)$$

$$w_2 = \frac{w_\infty}{\kappa + 1} \lambda \sin 2\sigma$$

$$z = \rho v_n v_s$$

$$\kappa = \text{adiabatic exponent}$$

$$\lambda = 1 - \frac{1}{M_\infty^2 \sin^2 \sigma}$$

$$\varphi = \frac{p}{\rho^\kappa} = \text{entropy function} \qquad (\varphi_\infty = 1)$$

$$\varphi_b = \frac{4\kappa w_\infty^{-2\kappa}}{\kappa^2 - 1} \left(\frac{\kappa - 1}{\kappa + 1}\right)^\kappa \left[\frac{w_\infty^2}{1 - w^2_\infty} - \frac{(\kappa - 1)^2}{4\kappa} \right]$$

$$\rho = \tau \varphi^{-1/(\kappa - 1)} = \text{density}$$

$$\tau = (1 - w^2)^{1/(\kappa - 1)}$$

Notes: (1) All velocities are nondimentionalised by the maximum velocity of the gas. All the pressures and densities are nondimentionalised by their corresponding stagnation values *in front* of the shock wave. (2) Subscripts b, δ, ∞, refer to the conditions at the body, at the shock, and in the undisturbed flow, respectively.

SYMBOLS USED IN THE FINAL SYSTEM OF DIFFERENTIAL EQUATIONS

$$A = (1 - w_\delta^2)^m \frac{2w_\infty}{\kappa + 1} \left[\left(P \sin \theta - Q \cos \theta\right) \left(\cos 2\sigma + \frac{1}{M_\infty^2 \sin^2 \sigma}\right) - \left(P \cos \theta + Q \sin \theta\right) \sin 2\sigma \right]$$

$$B = (1 - v_b^2)^m \left(1 - \frac{v_b^2}{a_\kappa^2}\right)$$

$$C = \frac{1}{\delta} (\tau_b v_b - \tau_\delta v_{s\delta})$$

$$D = - h_\delta \left(\frac{2}{\delta} + \frac{1}{R}\right)$$

$$E = \delta\left\{\frac{\kappa+1}{2M_\infty^2}\frac{\rho_\infty v_{n\delta} v_{s\delta}\sin 2\sigma}{\left(\frac{\kappa-1}{2}\sin^2\sigma+\frac{1}{M_\infty^2}\right)^2} + \left[\left(v_{s\delta}\sin\theta - v_{n\delta}\cos\theta\right)\sin 2\sigma\right.\right.$$

$$\left.\left. + (v_{s\delta}\cos\theta + v_{n\delta}\sin\theta)(\cos 2\sigma + \frac{1}{M_\infty^2\sin^2\sigma})\right]\rho\delta\ \frac{2w_\infty}{K+1}\right\}$$

$$F = -z_\delta$$

$$G = -\rho_\delta\left(2+\frac{\delta}{R}\right)\left[v_{n\delta}^2 + k(1-w_\delta^2)\right] +$$

$$2k\rho_b\left[\frac{\delta}{2R}\left(1+\frac{v_b^2}{a_\kappa^2}\right) + \left(1 - v_b^2\right)\right]$$

$$L = \left(1+\frac{\delta}{R}\right)\tan\,(\sigma-\theta)$$

$$P = 1 - v_{n\delta}^2 - \frac{v_{s\delta}^2}{a_\kappa^2}$$

$$Q = \frac{2v_{s\delta}v_{n\delta}}{\kappa-1}$$

THEORETICAL BACKGROUND

THE MAIN SYSTEM OF ORDINARY DIFFERENTIAL EQUATIONS

In the (n, s) coordinate system, as shown in Fig. 1, the continuity equation and the condition of entropy conservation along the streamlines, yield after suitable transformations the following formula:

$$\frac{\partial t}{\partial s} + \frac{\partial}{\partial n}\left[h\left(1+\frac{n}{R}\right)\right] = 0 \tag{1}$$

Similarly, the continuity equation and the momentum equation in the n-direction yield:

$$\frac{\partial z}{\partial s} + \frac{\partial}{\partial n}\left[H\left(1+\frac{n}{R}\right)\right] = \frac{g}{R} \tag{2}$$

Using now the method of integral relations in order to replace the equations obtained by an equivalent system of ordinary differential equations, one can choose either the so-called "schema 1" or "schema 2" of this method

[7]. In the first schema, some of the functions appearing in Eqs. (1) and (2) are approximated by polygons or polynomials in n, so that s is the only remaining independent variable. The reverse is true for the second schema.

The degree of the polynomial or the number of the polygon sides is called the order of the approximation.

The choice of the schema depends obviously on the behaviour of the functions involved, and also on the aim of the computation. If, e.g., in a particular case a more detailed information in the s-direction is desired, the first schema is more advisable.

We are interested mainly in the changes in velocity, pressure, and density distributions on the body, caused by the changes of the angle of attack, and the first schema seems to fit our purpose better. We also confine ourselves to the first approximation, which gives anyway sufficient accuracy in the hypersonic range of speeds [8]. So we approximate linearly the three functions g, t, and z:

$$g = \left(\frac{n}{\delta}\right)\left(g_\delta - g_b\right) + g_b \tag{3}$$

$$t = \left(\frac{n}{\delta}\right)\left(t_\delta - t_b\right) + t_b \tag{4}$$

$$z = \left(\frac{n}{\delta}\right)z_\delta \qquad (z_b = 0) \tag{5}$$

in the region between the body (subscript b) and the shock (subscript δ). Making use of the energy equation,

$$\left(\frac{p}{\rho}\right) + w^2 = 1 \tag{6}$$

and of the approximating formulas (3)–(5), and also taking into account some purely geometric considerations as well as the Rankine-Hugoniot conditions for the shock, we obtain the following system of three oridnary differential equations:

$$\frac{d\delta}{ds} = L \tag{7}$$

$$\frac{d\delta}{ds} = \frac{G - FL}{E} \tag{8}$$

$$\frac{dv_b}{ds} = \frac{1}{B}\left[D - CL - \frac{A}{E}\left(G - FL\right)\right] \tag{9}$$

The right-hand sides depend only on δ, σ, v_b and s.*

* The body shape is known, so R and θ are known fuctions of s.

COMPUTATION OF THE STREAMLINES

In order to evaluate σ_{int}, the stagnation streamline must be computed in the asymmetric case, and the corresponding fourth equation must be included in the system (7)–(9). It stems from the condition of stream-function conservation along the streamline, and its final form is

$$\frac{dn}{ds} = \frac{v_n}{v_s}\left(1 + \frac{n}{R}\right) \tag{10}$$

The velocity components v_n, v_s are calculated consistently with the approximating formulas (3)–(5) in the following manner. First, the values g, t, and z are computed in the point considered. Then v_n, v_s, ρ, p are sought as roots of the system of four algebraic equations:

$$t = v_s(1 - v_n^2 - v_s^2)^{1/(\kappa-1)}$$

$$z = \rho v_n v_s$$

$$g = \rho v_n^2 + kp$$

$$\frac{p}{\rho} = 1 - v_n^2 - v_s^2$$

This must be done by a suitable trial-and-error procedure.

THE INITIAL VALUES AND THE ADDITIONAL CONDITIONS

The solution to the problem in question are the three functions $v_b(s)$, $\delta(s)$, $\sigma(s)$. In order to obtain them, Eqs. (7)–(10) must be integrated starting from the properly chosen initial point and with proper initial values. The stagnation point seems to be the best starting point for the integration—at least in the present authors' opinion gained on the ground of rather negative results of a different approach.

Denoting x_0, δ_0, σ_0 the position of the stagnation point on the body (Fig. 1) and the (unknown) initial values of δ and σ, respectively, one may pose the initial value problem as follows:

$$\left\{ \begin{array}{ll} \text{at } x = x_0, \quad \text{where } s = 0: & v_b = 0 \\ & \delta = \delta_0 \\ & \sigma = \sigma_0 \\ & n = 0 \end{array} \right. \tag{11}$$

All the three values x_0, δ_0, σ_0 are unknown, so that three additional conditions must be imposed in order to evaluate them. The first two of these values express the requirement that the numerator and denominator of Eq. (9) must vanish simultaneously, so that the velocity slope remains finite:

$$D - CL - \frac{A}{E}\left(G - FL\right)\Big|_{v_b = +a_\kappa} = 0 \qquad (12)$$

$$D - CL - \frac{A}{E}\left(G - FL\right)\Big|_{v_b = -a_\kappa} = 0 \qquad (13)$$

The third condition stems from assumption (5) accepted at the beginning of this paper, which says

$$\sigma_{\text{int}} - \frac{\pi}{2} = 0 \qquad (14)$$

In concluding this section the assumption of the identity of the maximum entropy streamline and the stagnation streamline should be briefly discussed. This was first introduced by Mangler [18], whose method for solving the indirect problem applied only if this assumption was taken into account. If not, the method would lead "to an unlikely flow pattern."

In fact, the same assumption was made in all the papers [3–13] dealing with the symmetric case, but it does not cause any doubts there due to the symmetry of the flow investigated. It does not look so obvious, however, in the asymmetric case, and there was an attempt [14] to prove its correctness. The attempt was not quite successful [19,20] and recently Swigart published his work [19,20], which does not make any use of the assumption in question. Moreover, Swigart's results, obtained by the use of his method of solving the indirect problem, show a discrepancy between the maximum entropy streamline and the stagnation streamline.

In the opinion of the present authors, however, Swigart's very interesting results do not undermine too severely the assumption in question. First, the discrepancies are very small, and second, Swigart's method is not exact, so that the discrepancies might be caused by the truncation errors. In any case, at the present state of affairs, the identity of the maximum entropy and stagnation streamlines seems to be a justified *assumption*, which while not expressing the whole truth is at least a fair approximation.

SOME DETAILS OF THE NUMERICAL PROCEDURE

ERROR INDICATORS

The whole computing procedure consists of two main parts, i.e., (1) evaluation of the initial values x_0, δ_0, σ_0 by the use of the proper iteration technique; (2) computing of final results such as velocity and pressure distribution along the body.

The iteration process is based on the so-called "error indicators," i.e., certain quantities, which indicate the degree of accuracy of fulfilling the conditions (12)–(14).

The evaluation of the three error indicators E_1, E_2, E_3 chosen in the present work proceeds as follows.

In order to evaluate E_1, the system of three differential equations (7)–(9) is numerically integrated for a set of arbitrary initial values x_0, δ_0, σ_0 with a *positive* step* Δs, until the velocity modulus $|v_b|$ becomes equal to a prescribed quantity, a little lower than the critical velocity a_κ (Figs. 2*a* and 2*b*), or until the numerator $[D - CL - A/E(G - FL)]$ becomes equal to or less than zero (Fig. 2*c*). The integration pauses then and the error indicator E_1 is evaluated by the use of linear interpolation or extrapolation, as shown schematically in Fig. 2. In the schema according to Fig. 2*a*, the error indicator is assumed to be positive; in the schema shown on Figs. 2*b* and 2*c* it is assumed to be negative.

The same is true as far as evaluation of the second error indicator E_2 is concerned, only the integration is performed with a negative step $\Delta s < 0$.]

The definition of the third error indicators E_3 is in full accordance with the condition (14), i.e.,

$$E_3 = \sigma_{\text{int}} - \left(\frac{\pi}{2}\right) \tag{15}$$

In order to evaluate E_3, the system of *four* differential equations (7)–(10) is integrated numerically starting from the stagnation point (S in Fig. 1.), and the integration proceeds until the computed stagnation streamline reaches the shock wave:

$$n \geqslant \delta \tag{16}$$

The shock-wave angle σ_{int} in the intersection point (A in Fig. 1) of the streamline and of the shock is interpolated, and the evaluation of the third error indicator follows, according to formula (15).

It should be mentioned here, that in computations of the stagnation streamline n instead of s was used as the independent variable in the equations (7)–(10), for the sake of convenience.

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* The step was of the order 2.10^{-2}–4.10^{-2}, and the Runge-Kutta-Gill method was used in the integration.

ITERATION TECHNIQUE

The schema of computing the successive approximations of x_0, δ_0, and σ_0 is shown in the flow chart (Fig. 3). The iteration procedure turned out to be convergent, provided that the values of x_0, δ_0, and σ_0 used as the first approximation were not too far from the correct ones. As can be seen from the flow chart, the correct values of x_0, δ_0, and σ_0 are approached from both sides, and the iteration process terminates only when the difference between two approximated values, corresponding to two error indicators *of different sign*, is smaller than a prescribed value β.

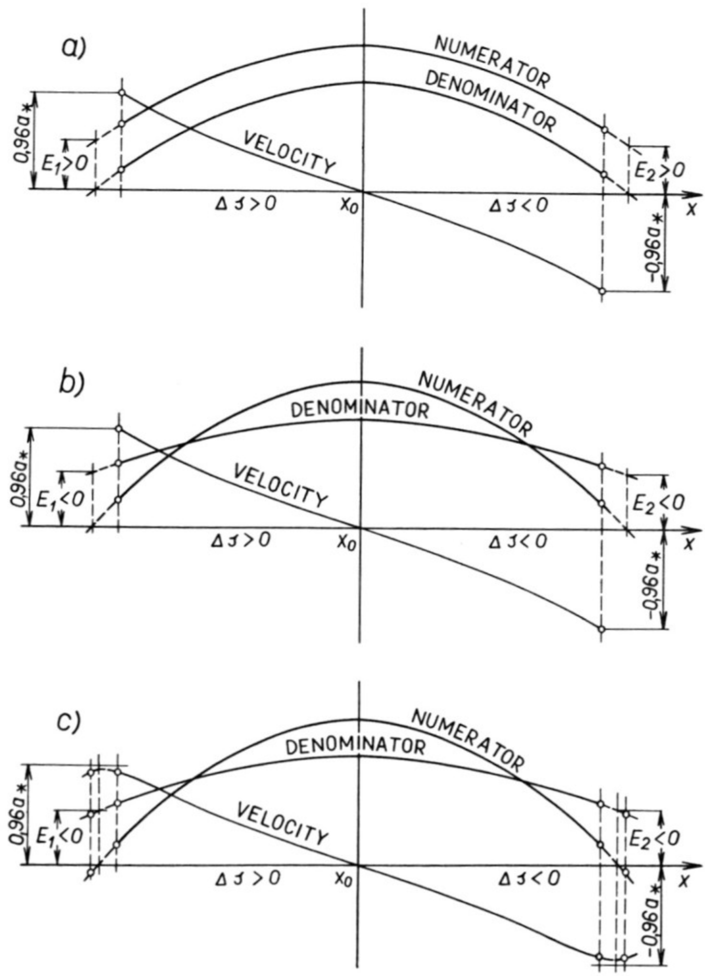

Figure 2. Evaluation of "error indicators."

It should be noted that x_0, δ_0, and σ_0 must be computed with great accuracy—i.e., at least to five significant numbers.

FINAL RESULTS

When the initial values x_0, δ_0, and σ_0 are computed with sufficient accuracy, the aim of the iteration procedure is achieved, and the computation and printing out of the final results may follow.

They consist of the functions $v_b(s)$, $p_b(s)$, $\rho_b(s)$, $\sigma(s)$, $\delta(s)$, $v_{s\delta}(s)$, $v_{n\delta}(s)$, $p\delta(s)$, $\rho\delta(s)$, $n(s)$, $x(s)$, as well as of the x,y-coordinates of the shock front and of the stagnation streamline.

The functions v_b, σ, δ, and n are computed by integration of the equations (7)–(10), and the rest of the above-mentioned functions is evaluated at each step of integration using the proper relations between them and v_b, σ, and δ. The relations in question are also given above.

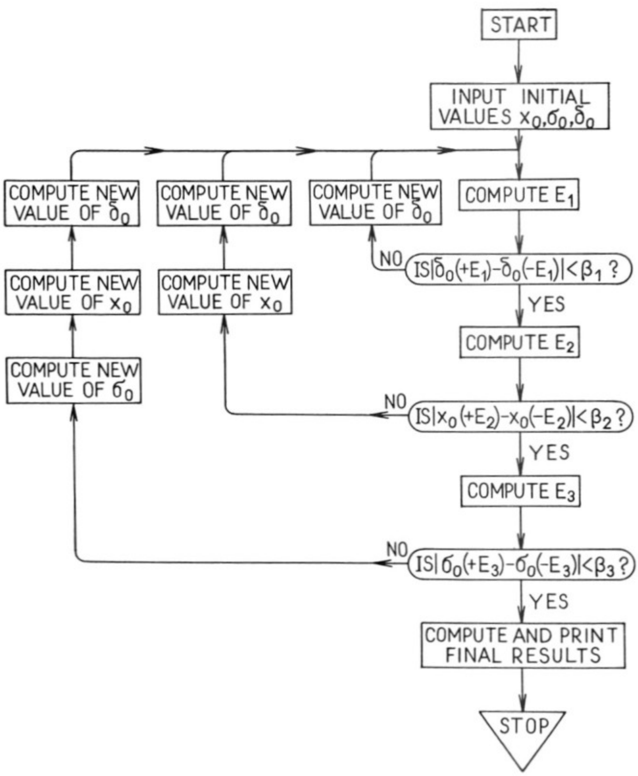

Figure 3. Flow chart for the calculating procedure.

The only difference between using the equations for calculation of the error indicators and for calculation of the final results consists in transition through the critical (sonic) points. This transition is immaterial when computing the error indicators, because the integration always pauses before the velocity v_b reaches the critical value. It becomes important, however, if the supersonic region in the vicinity of sonic lines has to be computed in order to permit extension of the flow-field calculations by the more exact method of characteristics. In our paper the transition across the body sonic point is performed in the following simple way. In the interval of about two steps "in front" of the sonic point, and about two steps "behind" it, the velocity gradient dv_b/ds is kept constant. In this manner the appearance of overflow in the sonic point, and the numerical inaccuracies in its vicinity are avoided.

RESULTS

The computations were performed on a GIER electronic computer for Mach number $M_\infty = 3$; for adiabatic exponent $\kappa = 1.4$; for a prolate elliptic profile of axes ratio $a/b = 4$ at five angles of attack $\alpha = 0°$; $1°$; $2.5°$; $5°$; $7.5°$.

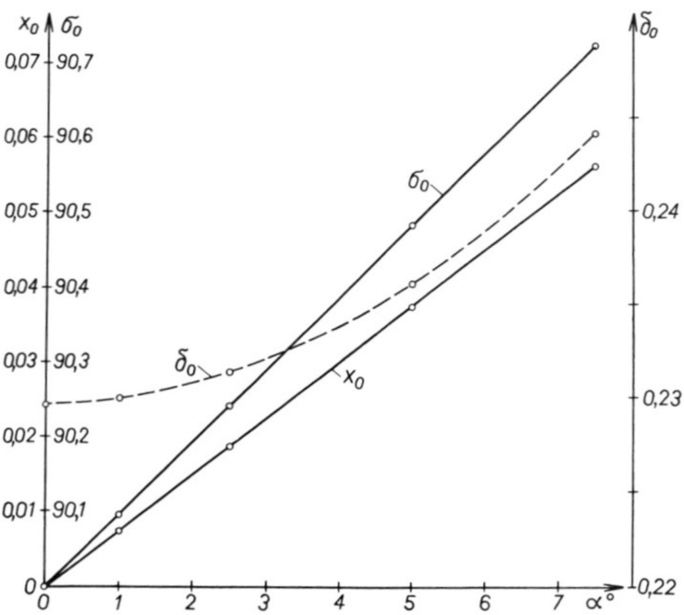

Figure 4. The initial values vs. the angle of attack.

The variation with α of initial values x_0, δ_0, σ_0 is shown in Fig. 4 and in Table 1. It is interesting to note, that σ_0 and x_0 vary with α almost lineraly.

Some of the remaining results are contained in Figs. 5–10, and in Tables 2 and 3.

It can be easily seen (Fig. 5) that the shape of the subsonic region as well as the positions of critical points both on the shock and on the body (Fig. 6) change very distinctly with the angle of attack.

TABLE 1

α	x_0	δ_0	σ_0
0°	0.000000	0.229896	90.0000
1°	0.007417	0.230190	90.0963
2.5°	0.018599	0.231514	90.2425
5°	0.037409	0.236252	90.4833
7.5°	0.056724	0.244289	90.7271

TABLE 2

($\alpha = 0^0$)

Body coordinates		Gas-dynamic quantities			Shock-wave coordinates and angle		
x	y	$v_b/a\kappa$	p_b	ρ_b	x_δ	y_δ	σ
0.000	0.000	0.000	0.328	0.328	0.000	−0.230	90.00
−0.040	0.003	0.124	0.325	0.326	−0.076	−0.227	85.35
−0.079	0.012	0.256	0.316	0.319	−0.151	−0.218	81.46
−0.116	0.027	0.390	0.300	0.308	−0.224	−0.205	78.54
−0.151	0.046	0.516	0.280	0.293	−0.295	−0.189	76.31
−0.184	0.068	0.626	0.259	0.277	−0.364	−0.171	74.32
−0.215	0.094	0.715	0.240	0.262	−0.431	−0.151	72.28
−0.244	0.121	0.786	0.223	0.249	−0.496	−0.129	70.22
−0.258	0.136	0.817	0.217	0.244	−0.528	−0.177	69.25
−0.272	0.150	0.846	0.210	0.239	−0.560	−0.105	68.30
−0.285	0.165	0.872	0.204	0.234	−0.591	−0.092	67.38
−0.297	0.181	0.896	0.199	0.229	−0.621	−0.079	66.49
−0.310	0.197	0.919	0.193	0.224	−0.652	−0.066	65.64
−0.322	0.213	0.940	0.188	0.220	−0.681	−0.052	64.83
−0.333	0.229	0.961	0.183	0.216	−0.711	−0.038	64.04
−0.345	0.245	0.981	0.178	0.212	−0.740	−0.024	63.27
−0.356	0.262	1.000	0.173	0.208	−0.769	−0.009	62.53
−0.367	0.279	1.019	0.169	0.204	−0.797	0.006	61.81
−0.377	0.296	1.037	0.164	0.200	−0.825	0.021	61.11
−0.388	0.313	1.054	0.160	0.197	−0.852	0.037	60.43

TABLE 3
(α = 7.5⁰)

Body coordinates		Gas-dynamic quantities			Shock-wave coordinates and angle		
x	y	$v_b/a\kappa$	p_b	ρ_b	x_δ	y_δ	σ
0.057	0.006	0.000	0.328	0.328	0.111	−0.232	90.73
0.017	0.001	0.119	0.326	0.326	0.034	−0.240	86.14
−0.023	0.001	0.252	0.316	0.320	−0.045	−0.242	81.92
−0.062	0.008	0.397	0.299	0.307	−0.124	−0.239	78.59
−0.100	0.020	0.543	0.275	0.289	−0.201	−0.231	76.10
−0.136	0.037	0.678	0.248	0.269	−0.279	−0.221	74.01
−0.170	0.058	0.788	0.222	0.249	−0.355	−0.209	71.78
−0.186	0.070	0.835	0.213	0.241	−0.392	−0.201	70.63
−0.202	0.082	0.876	0.203	0.233	−0.429	−0.193	69.43
−0.217	0.096	0.912	0.194	0.226	−0.466	−0.184	69.24
−0.232	0.109	0.947	0.186	0.219	−0.502	−0.174	67.05
−0.246	0.123	0.983	0.176	0.211	−0.538	−0.164	65.81
−0.260	0.138	1.009	0.171	0.206	−0.573	−0.153	64.73
−0.273	0.152	1.034	0.165	0.201	−0.608	−0.142	63.74
−0.286	0.168	1.056	0.160	0.196	−0.642	−0.130	62.82
−0.299	0.183	1.072	0.157	0.194	−0.676	−0.117	62.10
0.057	0.006	0.000	0.328	0.328	0.111	−0.232	90.73
0.095	0.018	−0.115	0.326	0.327	0.185	−0.218	94.91
0.131	0.035	−0.230	0.318	0.321	0.256	−0.200	98.22
0.166	0.055	−0.343	0.306	0.312	0.323	−0.180	100.67
0.198	0.079	−0.447	0.292	0.302	0.388	−0.157	102.57
0.228	0.105	−0.538	0.276	0.290	0.451	−0.133	104.26
0.256	0.133	−0.614	0.261	0.279	0.512	−0.108	105.93
0.283	0.163	−0.676	0.248	0.269	0.571	−0.081	107.60
0.308	0.194	−0.728	0.237	0.260	0.629	−0.053	109.22
0.332	0.227	−0.774	0.227	0.252	0.685	−0.024	110.75
0.343	0.243	−0.794	0.223	0.249	0.712	−0.009	111.46
0.354	0.260	−0.813	0.218	0.245	0.739	0.006	112.16
0.365	0.276	−0.832	0.214	0.242	0.766	0.021	112.83
0.376	0.293	−0.849	0.210	0.238	0.792	0.037	113.47
0.386	0.310	−0.865	0.206	0.235	0.818	0.053	114.09
0.396	0.328	−0.881	0.202	0.232	0.844	0.069	114.69
0.406	0.345	−0.896	0.199	0.229	0.869	0.085	115.26
0.416	0.363	−0.910	0.195	0.226	0.894	0.101	115.81
0.425	0.380	−0.923	0.192	0.224	0.919	0.118	116.35
0.435	0.398	−0.936	0.189	0.221	0.944	0.134	116.87
0.444	0.416	−0.949	0.186	0.219	0.968	0.151	117.37
0.453	0.434	−0.962	0.183	0.216	0.992	0.168	117.85
0.462	0.452	−0.974	0.180	0.214	1.016	0.185	118.33
0.470	0.470	−0.986	0.177	0.211	1.040	0.202	118.79
0.479	0.488	−0.997	0.174	0.209	1.063	0.220	119.24
0.487	0.506	−1.009	0.171	0.206	1.086	0.237	119.69
0.495	0.524	−1.020	0.169	0.204	1.109	0.255	120.13
0.503	0.543	−1.031	0.166	0.202	1.132	0.272	120.55
0.511	0.561	−1.042	0.163	0.199	1.154	0.290	120.97
0.518	0.579	−1.047	0.162	0.198	1.177	0.308	121.36

Distributions of velocity components and gas-dynamic parameters along the stagnation streamline were also computed, but they are not included in the present paper because the calculation of the stagnation streamline as a whole is rather subject to the main purpose of computations. One feature of the results concerning the streamline should be mentioned, however. It turns out that the entropy is not constant along the stagnation streamline, as it should be, but reaches the maximum value only in both end points of the streamline (points A and S in Fig. 1). This discrepancy is due to the approximations (3)–(5) and it is unavoidable. It could be remedied only in a rather artificial and inconsistent manner, if one of these approximating formulas were abandoned and the condition of entropy conservation were used instead in calculation of velocity components on the streamline.

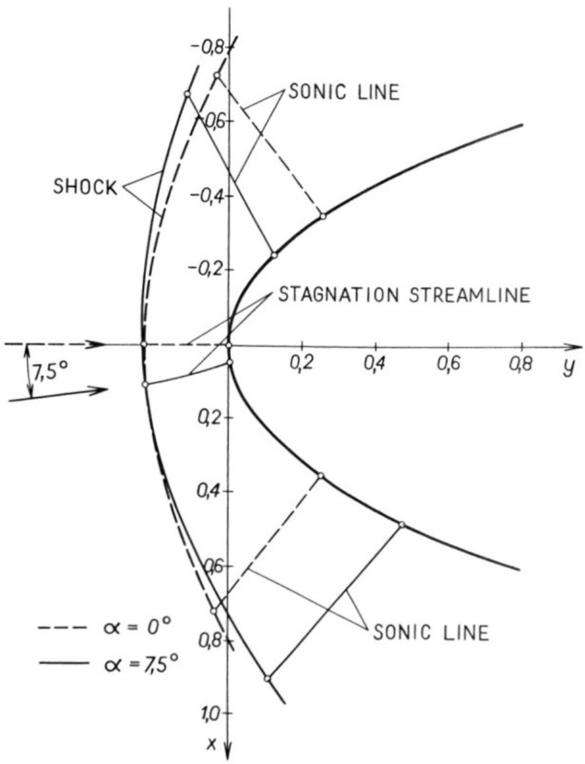

Figure 5. Shock shapes, sonic lines and stagnation streamlines for two angles of attack $\alpha = 0°$ and $= 7.5°$.

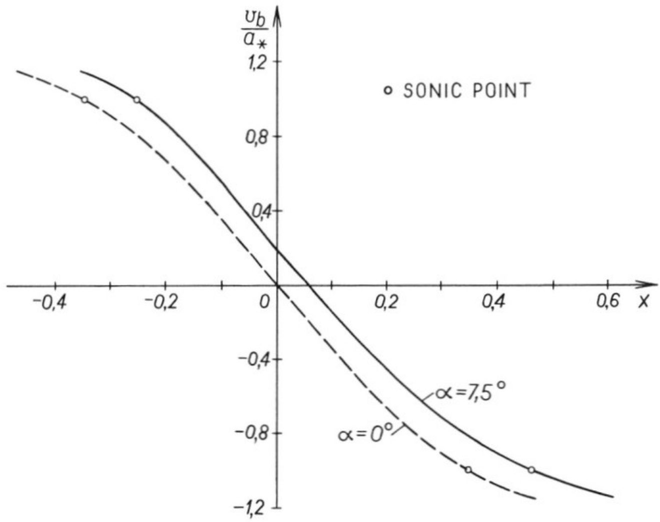

Figure 6. Velocity distribution.

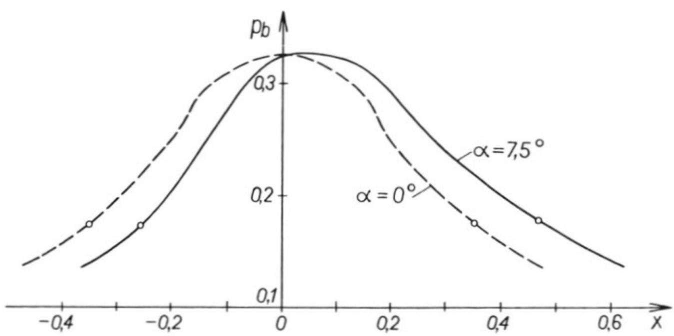

Figure 7. Pressure distribution.

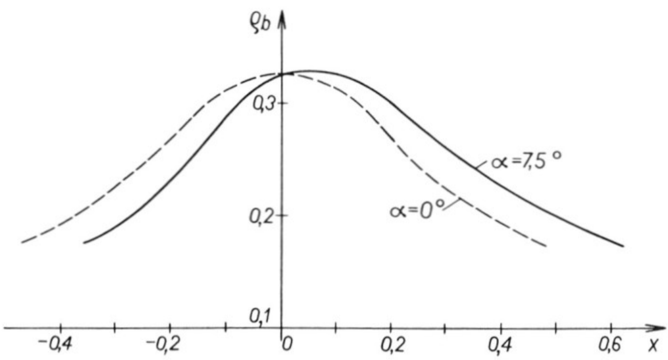

Figure 8. Density distribution.

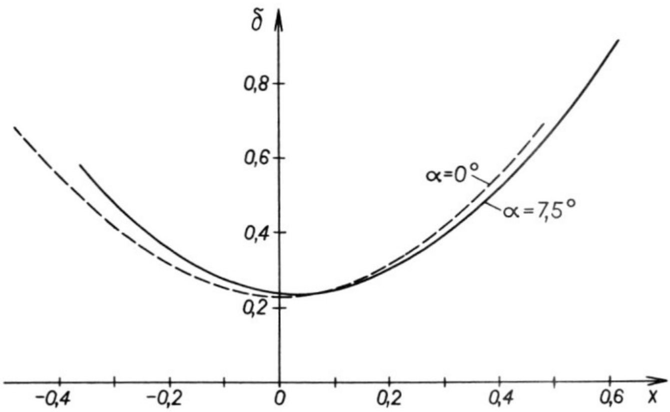

Figure 9. Shock standoff distance.

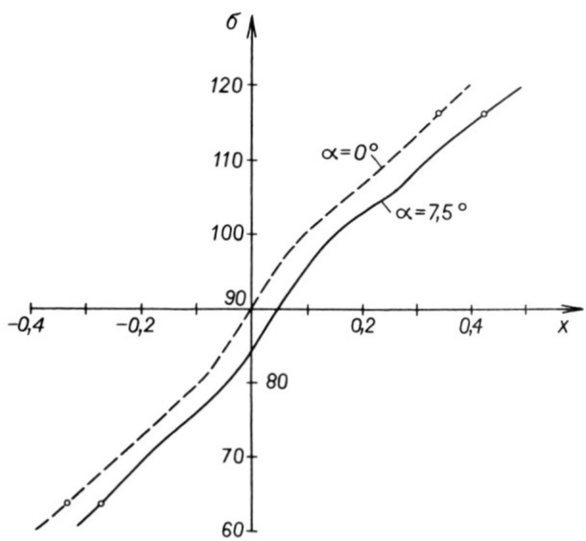

Figure 10. Shock-wave angle.

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COMMENTARY

J. P. GUIRAUD (*O.N.E.R.A., Chatillon-sur-Ragneux, France*): Est ce que la neassité que vous avez de faire appel à une condition inspirée par la conjecture de Managler n'est pas due au fait que la méthode ne permet pas de tracer avec précision la ligne de courant d'arrêt. En construisant cette ligne de courant a posteriori, ne ferait-il pas possible de remplacer la condition que vous avez pris par une condition de débit?

REPLY

The brief comments of Dr. Guiraud call for a rather lengthy explanation.

There exists an infinite number of "solutions" to the problem considered in the paper, each of them consisting of three functions for the velocity distribution along the body, the shock distance, and the shock angle, respectively, and each of them satisfying the conditions imposed in the two critical points. The method of integral relations itself does not contain (in the discussed case) any condition, either of physical or of mathematical nature, which could serve the purpose of telling which one of the "solutions" is the physically meaningful one. Therefore, an additional condition must be imposed. Its logical necessity has nothing to do with computation of streamlines.

On the other hand, the streamline pattern corresponding to each "solution" is unique: there exists only one manner of computing it within the frame of the method. Accordingly, the "solutions" differ also in the shape of the stagnation streamline, and in such a sense—and only in such a sense—could one say that the stagnation streamline is not determined *avec precision*. The meaning of this lack of precision is, however, obvious: it follows from the non-uniqueness of the solution as a whole.

The assumption of identity of the maximum entropy line and the stagnation streamline, accepted in the paper, can be applied in many different ways in order to select the proper solution.

The condition for the mass flow and the condition used in the present paper, are examples of two such possible ways. They are strictly equivalent, at least formally—the second one, however, being more convenient from the computational viewpoint.